

4.1 Further Differentiation

Question Paper

Course	CIEAS Maths
Section	4. Differentiation
Topic	4.1 Further Differentiation
Difficulty	Hard

Time allowed: 50
Score: /41
Percentage: /100

Question 1

Use an appropriate method to differentiate each of the following.

(i) $\sin 2x - e^{7x}$

(ii) $x^2 \ln x$

(iii) $\frac{\cos 3x}{\tan 2x}$

(iv) $\ln(\tan x)$

[8 marks]

Question 2

A curve has the equation $y = e^{-3x} + \ln x$, $x > 0$.

Show that the equation of the tangent to the curve at the point with x -coordinate 1 is

$$y = \left(\frac{e^3 - 3}{e^3} \right) x + \frac{4 - e^3}{e^3}$$

[6 marks]

Question 3

For $y = \ln(ax^n)$, where $a > 0$ is a real number and $n \geq 1$ is an integer, show that

$$\frac{dy}{dx} = \frac{n}{x}$$

[3 marks]

Question 4

Find the gradient of the normal to the curve $y = 5 \cos(e^x - \frac{\pi}{2})$ at the point with x -coordinate 0. Give your answer correct to 3 decimal places.

[4 marks]

Question 5a

Differentiate with respect to x , simplifying your answers as far as possible:

(a) $(2 \sin 3x - \cos 3x)e^{6-x}$

[3 marks]

Question 5b

(b) $(x^2 - x)^2 \ln 5x$

[3 marks]

Question 6

By writing $y = \frac{f(x)}{g(x)}$ as $y = f(x)[g(x)]^{-1}$ and then using the product and chain rules, show that

$$\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

[3 marks]

Question 7a

Given that $x = \sec 7y$,

(a) Find $\frac{dy}{dx}$ in terms of y

[2 marks]

Question 7b

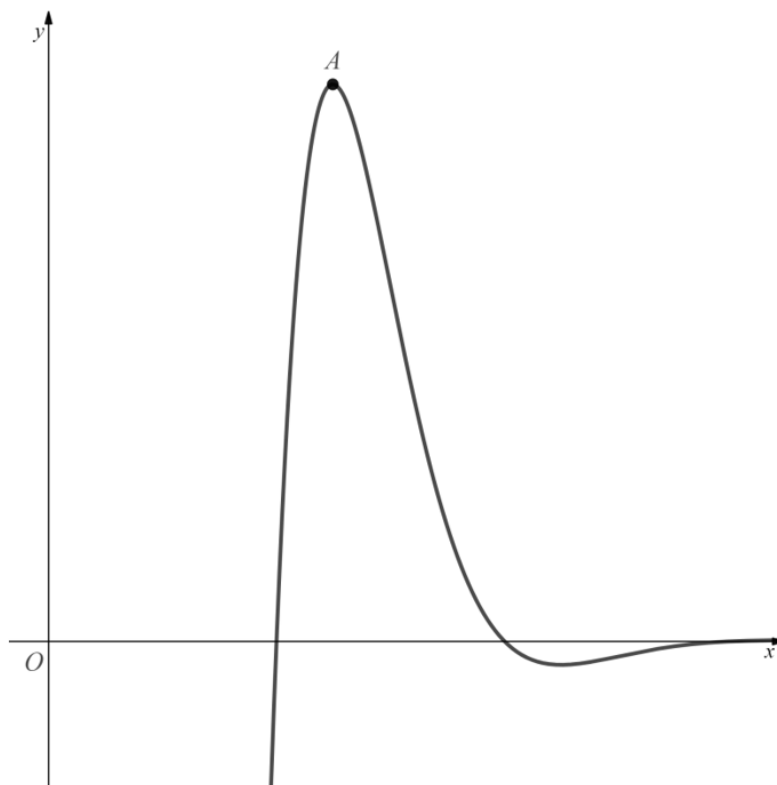
(b) Hence find $\frac{dy}{dx}$ in terms of x .

[4 marks]

Question 8

The diagram below shows part of the graph of $y = f(x)$, where $f(x)$ is the function defined by

$$f(x) = \frac{\sin x}{1 - e^x}, \quad x > 0$$



Point A is a maximum point on the graph.

Show that the x -coordinate of A is a solution to the equation

$$\frac{\cos x + e^x(\sin x - \cos x)}{e^{2x} - 2e^x + 1} = 0$$

[5 marks]

